TORUS ACTIONS, MORSE HOMOLOGY, AND THE HILBERT SCHEME OF POINTS ON AFFINE SPACE

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We formulate a conjecture on actions of the multiplicative group. In short, if the multiplicative group $\mathbf{G}_m$ acts on a quasi-projective scheme U such that U is attracted as t approaches 0 in $\mathbf{G}_m$ to a closed subset Y in U, then the inclusion from Y to U should be an $\mathbf{A}^1$-homotopy equivalence. This would be useful if true, since actions of the multiplicative group occur everywhere in algebraic geometry. We prove several partial results. The proofs use an analog of Morse theory for singular varieties. We give an application to the Hilbert scheme of points on affine space $\mathbf{A}^n$.